

U.C. MATH BOWL 2017

LEVEL III — Session 2

Instructions: Write your answers in the blue book provided. Remember that even correct answers without explanation may not receive much credit and that partially correct answers that show careful thinking and are well explained may receive many points.

Have Fun!

1. In the addition shown each letter represents a distinct digit. Can you find ways to assign digits to letters so that the addition is correct?

$$\begin{array}{r} \text{O N E} \\ + \text{O N E} \\ \hline \text{T W O} \end{array}$$

2. The mean annual income of 10 people is \$40,000. Four of the ten get equal pay raises. The recalculated mean annual income is now \$43,000. What was the amount of each raise?
3. Ben runs around the track at a pace of 1 lap every 80 seconds. Sally runs on the track in the opposite direction. The two meet every 30 seconds. How long does it take Sally to run each lap around the track?
4. Consider all the triangles having all three sides of integer length. Are there more such triangles with perimeter 11 or with perimeter 12? Justify your answer.
5. There is an imaginary island called the Island of Knights and Knaves. On this island, there are people called knights, who always tell the truth, and people called knaves, who always lie. The two types are indistinguishable by sight. On a vacation to this island, you meet two inhabitants: Bill and Bob. Bob says "We are both the same kind, either both knights or both knaves", but Bill says "We are both different kinds. One of us is a knave and the other is a knight" Who is the knight and who is the knave?